

12.1

Give a geometric description of the set of points in space whose coordinates satisfy the given pair of equations.

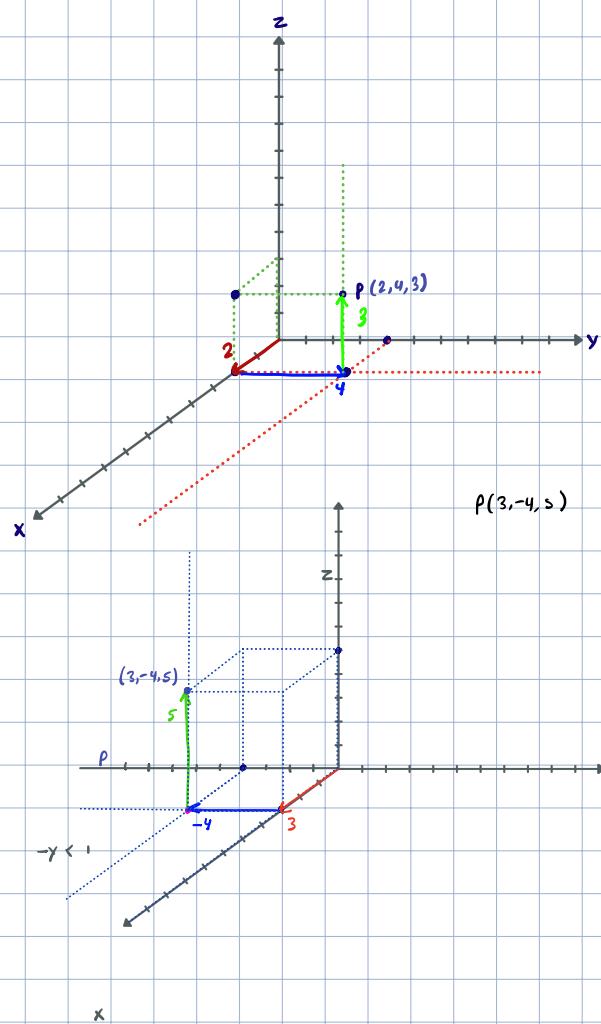
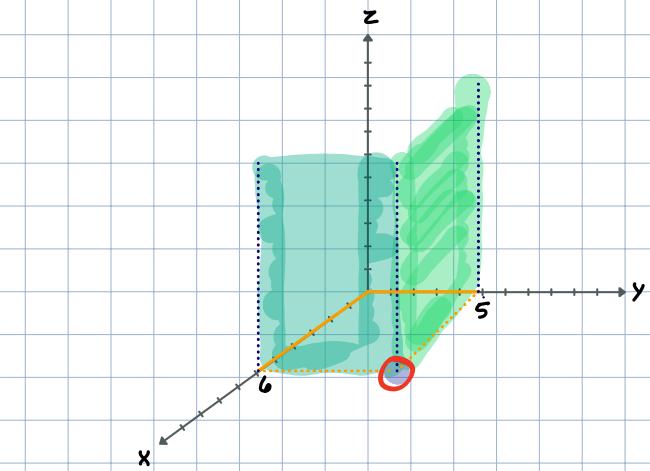
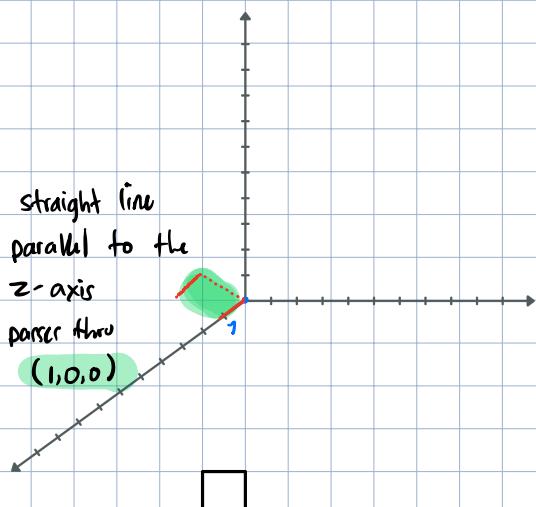
$$1. \ x = 6, \ y = 5$$

The point in space is $(6, 5, 0)$ since no z coordinate, $z = 0$.

The distance of the point $(6, 5, 0)$ from the origin is,

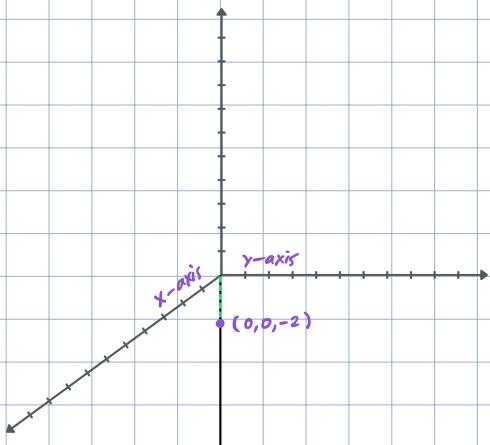
$$\begin{aligned} d &= \sqrt{6^2 + 5^2 + 0^2} \\ &= \sqrt{36 + 25} \\ &= \sqrt{61} \end{aligned}$$

$$2. \ x = 1, \ y = 0$$



$$3. \quad x^2 + y^2 = 4, \quad z = -2$$

The circle with center $(0,0,-2)$ and radius 2 parallel to the xy -plane.

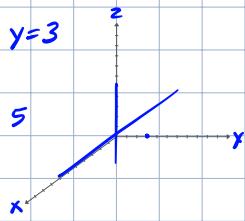


$$4. \quad x^2 + z^2 = 25, \quad y = 3$$

$$(0, 3, 0)$$

this represents a circle of radius 5 centered at $y=3$

\therefore the circle with center $(0,3,0)$ and radius 5 parallel to the xz -plane



$$5. \quad x^2 + y^2 + z^2 = 9, \quad x = 0$$

this represents a circle centered at $(0,0,0)$ with a radius 3.

$$x^2 + y^2 + (z+4)^2 = 100, \quad z = 2$$

that is $x^2 + y^2 + (z+4)^2 = 100$

$$x^2 + y^2 + 6^2 = 100$$

$$x^2 + y^2 = 64$$

$$\therefore x^2 + y^2 + (z+4)^2 = 100 \text{ and } z = 2$$

represent a circle of radius 8

centered at $(0,0,2)$ parallel to the xy -plane

RECITATION

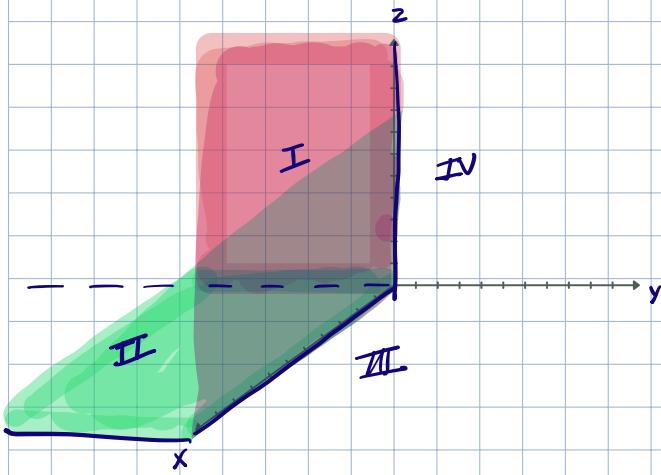
$$\boxed{x^2 + y^2 = 4, \quad z = y}$$

$$x^2 + y^2 = 16, \quad z = x$$

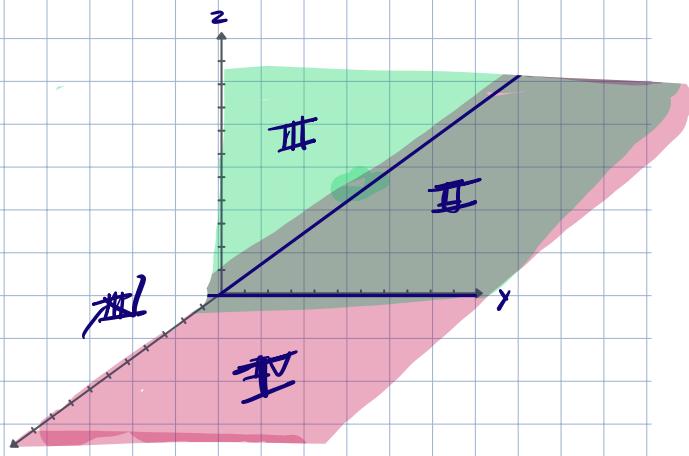
Describe the set of points in space whose coordinates satisfy the given combination of equations and inequalities.

a. $x \geq 0, y = 0, z \geq 0$

the first quadrant of
the xz -plane



b. $x \leq 0, y \geq 0, z = 0$



Find the distance between points P_1 and P_2

$$P_1(4, 2, 5) \quad P_2(8, 0, 9)$$

$$P_1P_2 = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$$

$$P_1P_2 = \sqrt{(8-4)^2 + (0-2)^2 + (9-5)^2}$$

$$= \sqrt{16+4+16}$$

$$= \sqrt{36}$$

$$= 6$$

$$P_1(0, 0, 0) \quad P_2(-5, 5, -4)$$

$$P_1P_2 = \sqrt{(-5-0)^2 + (5-0)^2 + (-4-0)^2}$$

$$= \sqrt{25+25+16}$$

$$= \sqrt{66}$$

Find the distance from the point $(1, 5, -3)$ to the

- a. xy -plane b. yz -plane c. xz -plane

$$(x_1, y_1, z_1) = (1, 5, -3)$$

a. xy -plane = $(1, 5, 0)$ \therefore the distance from $(1, 5, -3)$ to the xy -plane is $|z_1| = 3$

b. yz -plane = $(0, 5, -3)$ \therefore the distance from $(1, 5, -3)$ to the yz -plane is $|x_1| = 1$

c. xz -plane = $(1, 0, -3)$ \therefore the distance from $(1, 5, -3)$ to the xz -plane is $|y_1| = 5$

find the distance from the point $(-3, -5, -6)$

- a. xy-plane b. yz-plane c. xz-plane

a. xy-plane $(-3, -5, 0) \therefore d = |z| = 6$

b. yz-plane $(0, -5, -6) \therefore d = |x| = 3$

c. xz-plane $(-3, 0, -6) \therefore d = |y| = 5$

describe the plane perpendicular to each of the following axes at the given points with a single equation or a pair of equations.

a. the z-axis at $(6, -7, -6)$

b. the x-axis at $(4, -1, 4)$

c. the y-axis at $(-4, 3, 6)$

Describe the given set with a single equation or a pair of equations.

The circle of radius 6 centered at $(0, 9, 0)$ and lying in:

$$(x-0)^2 + (y-9)^2 + (z-0)^2 = 6^2$$

a. xy-plane

$$x^2 + (y-9)^2 + z^2 = 36, \quad z = 0$$

$$x^2 + (y-9)^2 = 36, \quad z = 0$$

b. yz-plane

$$x^2 + (y-9)^2 + z^2 = 36, \quad x = 0$$

$$(y-9)^2 + z^2 = 36, \quad x = 0$$

c. plane $y=9$

$$x^2 + (y-9)^2 + z^2 = 36, \quad y = 9$$

$$x^2 + (9-9)^2 + z^2 = 36, \quad y = 9$$

$$x^2 + z^2 = 36, \quad y = 9$$

Describe the given set with a single equation or a pair of equations.

the line through the point $(-4, 2, -2)$ parallel to the:

a. x-axis b. y-axis c. z-axis

$$y=2, z=-2 \quad x=-4, z=-2 \quad x=-4, y=2$$

Write an inequality to describe the following set.

the slab bounded by the planes $x=0$ and $x=3$ (planes included)

$$0 \leq x \leq 3$$

Write an inequality to describe the following set.

a. the interior of the sphere of radius 7 centered at the point $(7, -5, -6)$

b. the exterior of the sphere of radius 7 centered at the point $(7, -5, -6)$

$$(x-7)^2 + (y-(-5))^2 + (z-(-6))^2 = 7^2$$

a. $(x-7)^2 + (y+5)^2 + (z+6)^2 < 49$

b. $(x-7)^2 + (y+5)^2 + (z+6)^2 > 49$

find the center and radius of the sphere.

$$(x+5)^2 + y^2 + (z-5)^2 = 28$$

$$(x-x_0)^2 + (y-y_0)^2 + (z-z_0)^2 = a^2$$

$$\therefore (x_0, y_0, z_0) = (-5, 0, 5) \text{ Center}$$

$$\text{radius } a^2 = 28$$

$$\sqrt{a^2} = \sqrt{28} \rightarrow a = 2\sqrt{7} \text{ radius}$$

find the center and radius of the sphere.

$$x^2 + y^2 + z^2 - 2x - 2z = 0$$

$$(x - x_0)^2 + (y - y_0)^2 + (z - z_0)^2 = a^2$$

$$(x^2 - 2x) + y^2 + (z^2 - 2z) = 0$$

$$(x^2 - 2x + (-1)^2) + y^2 + (z^2 - 2z + (-1)^2) = 0 + (-1)^2 + (-1)^2$$

$$(x^2 - 1)^2 + y^2 + (z^2 - 1)^2 = 2 \quad \sqrt{a^2} = \sqrt{2}$$

$$x_0 = 1 \quad y_0 = 0 \quad z_0 = 1 \quad a = \sqrt{2}$$

Determine the equation for the sphere whose center and radius are given

Center	Radius
(1, 4, 3)	$\sqrt{18}$

Equation of the sphere (x_0, y_0, z_0)

$$(x - x_0)^2 + (y - y_0)^2 + (z - z_0)^2 = a^2$$

$$(x - 1)^2 + (y - 4)^2 + (z - 3)^2 = (\sqrt{18})^2$$

$$(x - 1)^2 + (y - 4)^2 + (z - 3)^2 = 18$$

Let $\vec{v} = \langle 2, -5 \rangle$. find the component form and magnitude (length) of the vector $4\vec{v}$.

Multiply a vector $\vec{v} = \langle v_1, v_2 \rangle$ by a scalar k by multiplying each component by the scalar.

$$k\langle v_1, v_2 \rangle = \langle kv_1, kv_2 \rangle$$

$$\begin{aligned} 4\vec{v} &= 4\langle 2, -5 \rangle \\ &= \langle 4(2), 4(-5) \rangle \\ &= \langle 8, -20 \rangle \end{aligned}$$

Magnitude or length of a vector $\vec{v} = \langle v_1, v_2 \rangle$ is the non-negative number $|\vec{v}| = \sqrt{v_1^2 + v_2^2}$

$$|4\vec{v}| = \sqrt{8^2 + (-20)^2} \rightarrow \text{Component } \langle 8, -20 \rangle$$

$$\text{magnitude } 4\sqrt{29}$$

Find a formula for the distance from the point $P(x, y, z)$ to each of the three axes.

a. Find the distance to the x -axis.

b. Find the distance to the y -axis.

c. Find the distance to the z -axis.

a. The distance of $P(x, y, z)$ from the x -axis is,

$$\begin{aligned} D &= \sqrt{(x - x)^2 + (y - 0)^2 + (z - 0)^2} \\ &= \sqrt{y^2 + z^2} \end{aligned}$$

b. The distance of $P(x, y, z)$ from the y -axis is,

$$\begin{aligned} D &= \sqrt{(x - 0)^2 + (y - y)^2 + (z - 0)^2} \\ &= \sqrt{x^2 + z^2} \end{aligned}$$

c. The distance of $P(x, y, z)$ from the z -axis is,

$$\begin{aligned} D &= \sqrt{(x - 0)^2 + (y - 0)^2 + (z - z)^2} \\ &= \sqrt{x^2 + y^2} \end{aligned}$$

Let $\vec{v} = \langle 3, -2 \rangle$. find the component form and magnitude (length) of the vector $6\vec{v}$.

$$\begin{aligned} 6\vec{v} &= 6\langle 3, -2 \rangle \\ k\langle v_1, v_2 \rangle &= \langle kv_1, kv_2 \rangle \\ &= \langle 18, -12 \rangle \end{aligned}$$

$$\begin{aligned} |6\vec{v}| &= \sqrt{18^2 + (-12)^2} \quad |\vec{v}| = \sqrt{v_1^2 + v_2^2} \\ &= \sqrt{468} \\ &= 6\sqrt{13} \end{aligned}$$

$$\text{Component } \langle 18, -12 \rangle$$

$$\text{magnitude } \sqrt{468}$$

$$\text{let } \vec{U} = \langle 5, -4 \rangle \text{ and } \vec{J} = \langle -2, -3 \rangle$$

find the component and the magnitude of $\vec{U} + \vec{J}$.

add vectors \vec{U} and \vec{J}

$$\text{magnitude of } \vec{U} + \vec{J} = \langle 3, -7 \rangle$$

$$\vec{U} + \vec{J} = \langle 5, -2 \rangle + \langle -4, -3 \rangle$$

$$= \langle 5 - 2, -4 - 3 \rangle$$

$$\vec{U} + \vec{J} = \langle 3, -7 \rangle \text{ component}$$

$$|\vec{U} + \vec{J}| = \langle 3, -7 \rangle \quad |\vec{V}| = \sqrt{V_1^2 + V_2^2}$$

$$= \sqrt{3^2 + (-7)^2}$$

$$= \sqrt{58}$$

$$\vec{U} + \vec{J} = \langle 3, -7 \rangle$$

$$|\vec{U} + \vec{J}| = \sqrt{58}$$

find the component form of the vector \vec{PQ} where $P = (7, 1)$ and $Q(8, -2)$

$$\vec{J} = \langle J_1, J_2 \rangle \quad \vec{P}_1 \vec{P}_2 = \langle x_2 - x_1, y_2 - y_1 \rangle$$

$$\begin{aligned} J_1 &= x_2 - x_1 & J_2 &= y_2 - y_1 \\ &= 8 - 7 & &= -2 - 1 \\ &= 1 & &= -3 \end{aligned}$$

$$\vec{PQ} = \langle 1, -3 \rangle$$

find the component form of the vector from the point $A = (3, 6)$ to the origin.

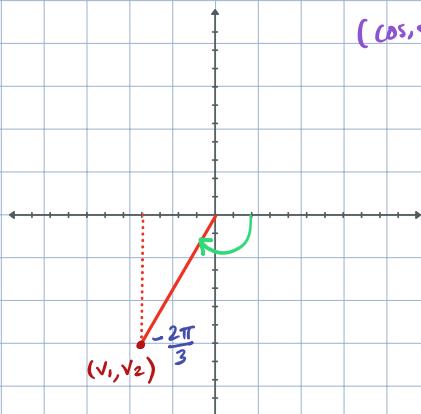
$$A = (3, 6), O = (0, 0)$$

$$\vec{AO} = (0 - 3)\vec{i} + (0 - 6)\vec{j}$$

$$= -3\vec{i} - 6\vec{j}$$

$$\text{component form of } \vec{AO} = \langle -3, -6 \rangle$$

Find the component form of the unit vector that makes an angle $\theta = -\frac{2\pi}{3}$ with the positive x-axis.



$$(\cos, \sin)$$

Since the length = 1

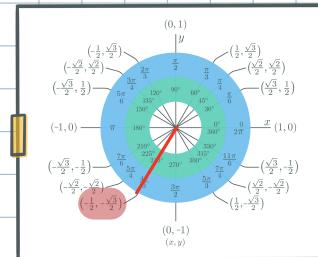
x coordinate

$$x = R \cos \theta$$

$$1 \text{ for } R, -\frac{2\pi}{3} \text{ for } \theta$$

$$= (1) \cos \left(-\frac{2\pi}{3}\right)$$

$$= -\frac{1}{2}$$



\therefore Component form is,

$$y = R \sin \theta$$

$$= (1) \sin \left(-\frac{2\pi}{3}\right)$$

$$= -\frac{\sqrt{3}}{2}$$

$$\langle -\frac{1}{2}, -\frac{\sqrt{3}}{2} \rangle$$

Express the vector $\vec{P_1P_2}$ in the form $\vec{v} = v_1\mathbf{i} + v_2\mathbf{j} + v_3\mathbf{k}$

if $P_1(7, -6, 6)$ and $P_2(8, 2, 5)$

$$\begin{aligned}\vec{P_1P_2} &= \langle (8-7)\mathbf{i} + (2-(-6))\mathbf{j} + (5-6)\mathbf{k} \rangle \quad \vec{P_1P_2} = \langle x_2-x_1, y_2-y_1, z_2-z_1 \rangle \\ &= \langle 1\mathbf{i} + 8\mathbf{j} + (-1)\mathbf{k} \rangle\end{aligned}$$

Express the following vector in the form $\vec{v} = v_1\mathbf{i} + v_2\mathbf{j} + v_3\mathbf{k}$

$3\vec{U} - \vec{V}$ if $\vec{U} = \langle 5, 6, -1 \rangle$ and $\vec{V} = \langle 4, -6, 2 \rangle$

$$\vec{U} = \langle 5, 6, -1 \rangle, \quad \vec{V} = \langle 4, -6, 2 \rangle$$

$$\text{so, } 3\vec{U} - \vec{V} = 3(5, 6, -1) - (4, -6, 2)$$

$$= (15, 18, -3) - (4, -6, 2)$$

$$= (11, 24, -5)$$

$$3\vec{U} - \vec{V} = 11\mathbf{i} + 24\mathbf{j} + (-5)\mathbf{k}$$

Express the vector $-12\mathbf{i} + 4\mathbf{j} - 6\mathbf{k}$ as a product of its length and direction.

$$\begin{aligned}\text{length} &= \sqrt{(-12)^2 + 4^2 + (-6)^2} \quad |\vec{v}| = \sqrt{v_1^2 + v_2^2 + v_3^2} \\ &= \sqrt{144 + 16 + 36} \\ &= \sqrt{196} = 14\end{aligned}$$

$$\text{direction} = \frac{-12\mathbf{i} + 4\mathbf{j} - 6\mathbf{k}}{14}$$

$$= -\frac{12}{14}\mathbf{i} + \frac{4}{14}\mathbf{j} - \frac{6}{14}\mathbf{k}$$

$$= 14 \left[-\frac{12}{14}\mathbf{i} + \frac{4}{14}\mathbf{j} - \frac{6}{14}\mathbf{k} \right]$$

$$= 14 \left[-\frac{6}{7}\mathbf{i} + \frac{2}{7}\mathbf{j} - \frac{3}{14}\mathbf{k} \right]$$

Express the vector $5\mathbf{k}$ as a product of its length and direction.

$$5\mathbf{k} = 0\mathbf{i} + 0\mathbf{j} + 5\mathbf{k}$$

$$\begin{aligned}\text{length} : \sqrt{0^2 + 0^2 + 5^2} \quad \text{Direction: } \frac{1}{5}(5\mathbf{k}) &= \mathbf{k} \\ &= \sqrt{25} \\ &= 5\end{aligned}$$

Find a vector of magnitude 3 in the direction of $\vec{v} = 24\mathbf{i} - 10\mathbf{k}$

$$|\vec{v}| = \sqrt{24^2 + 0^2 + (-10)^2}$$

$$= \sqrt{576 + 100}$$

$$= \sqrt{676}$$

$$= 26$$

direction



$$\frac{\vec{v}}{|\vec{v}|} = \frac{24}{26}\mathbf{i} + \frac{0}{26}\mathbf{j} + \left(\frac{-10}{26}\right)\mathbf{k}$$

$$= 3 \left(\frac{24}{26}\mathbf{i} + \left(\frac{-10}{26}\right)\mathbf{k} \right)$$

$$|\vec{v}| \frac{\vec{v}}{|\vec{v}|}$$

$$= \frac{72}{26}\mathbf{i} + \left(-\frac{30}{26}\right)\mathbf{k}$$

$$= \frac{36}{13}\mathbf{i} + \left(-\frac{15}{13}\right)\mathbf{k}$$

for the points $P_1(-1, 4, 3)$ and $P_2(2, 5, 0)$

find the direction $\vec{P_1P_2}$ and the midpoint of line segment P_1P_2 .

$$\vec{P_1P_2} = \langle x_2 - x_1, y_2 - y_1 \rangle$$

$$\begin{aligned} \vec{P_1P_2} &= (2 - (-1))\mathbf{i} + (5 - 4)\mathbf{j} + (0 - 3)\mathbf{k} \\ &= 3\mathbf{i} + 1\mathbf{j} + (-3)\mathbf{k} \end{aligned}$$

$$\text{midpoint } M = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}, \frac{z_1 + z_2}{2} \right)$$

$$M = \left(\frac{-1 + 2}{2}, \frac{4 + 5}{2}, \frac{3 + 0}{2} \right)$$

$$= \left(\frac{1}{2}, \frac{9}{2}, \frac{3}{2} \right) \quad \underline{\text{Midpoint}}$$

$$\frac{\vec{v}}{|\vec{v}|} = \frac{3\mathbf{i} + 1\mathbf{j} + (-3)\mathbf{k}}{\sqrt{3^2 + 1^2 + (-3)^2}}$$

$$|\vec{v}| = \sqrt{v_1^2 + v_2^2}$$

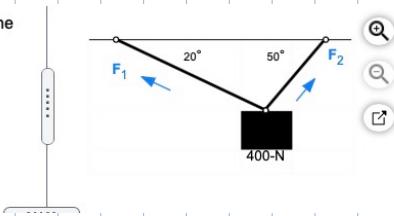
$$= \frac{3\mathbf{i} + 1\mathbf{j} + (-3)\mathbf{k}}{\sqrt{9 + 1 + 9}}$$

$$= \frac{3\mathbf{i} + 1\mathbf{j} + (-3)\mathbf{k}}{\sqrt{19}}$$

$$= \frac{3}{\sqrt{19}}\mathbf{i} + \frac{1}{\sqrt{19}}\mathbf{j} + \left(-\frac{3}{\sqrt{19}}\right)\mathbf{k}$$

direction

Consider a 400-N weight suspended by two wires as shown in the figure. Find the magnitude and components of the force vectors F_1 and F_2 .



$$F_1 = \langle -|F_1| \cos 20^\circ, |F_1| \sin 20^\circ \rangle \quad F_2 = \langle |F_2| \cos 50^\circ, |F_2| \sin 50^\circ \rangle$$

since $F_1 + F_2 = \langle 0, 400 \rangle$

$$-|F_1| \cos 20^\circ + |F_2| \cos 50^\circ = 0 \quad \textcircled{1}$$

$$|F_1| \sin 20^\circ + |F_2| \sin 50^\circ = 400 \quad \textcircled{2}$$

Solving for $|F_2|$ in $\textcircled{1}$ and substitute in second equation.

$$|F_2| = \frac{|F_1| \cos 20^\circ}{\cos 50^\circ} \quad \text{and} \quad |F_1| \sin 20^\circ + |F_1| \frac{\cos 20^\circ}{\cos 50^\circ} \sin 50^\circ = 400 \quad \frac{\sin \theta}{\cos \theta} = \tan \theta$$

$$|F_1| = \sin 20^\circ + \cos 20^\circ \tan 50^\circ = 400$$

$$|F_1| = \frac{400}{\sin 20^\circ + \cos 20^\circ \tan 50^\circ} \approx 273.6$$

$$|F_2| = \frac{400 \cos 20^\circ}{\sin 20^\circ \cos 50^\circ + \cos 20^\circ \sin 50^\circ} = 400 \quad |F_2| = \frac{|F_1| \cos 20^\circ}{\cos 50^\circ}$$

The force vectors are:

$$F_1 = \langle -|F_1| \cos 20^\circ, |F_1| \sin 20^\circ \rangle \approx \langle -257.1, 93.6 \rangle$$

$$F_2 = \langle |F_2| \cos 50^\circ, |F_2| \sin 50^\circ \rangle \approx \langle 257.1, 306.4 \rangle$$